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## General Remarks on Spectral Entropy vs. Statistical Entropy

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## Short Communication

### General Remarks on Spectral Entropy vs. Statistical Entropy<sup>1</sup>

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#### Abstract

Crepeau and Herzel [1] (CH) have recently compared the spectral entropy of Powell and Percival [2] with the standard statistical (Boltzmann–Gibbs–Shannon) entropy in three simple physical systems. Here we compare and contrast these two entropies in a more general way by considering their values for an arbitrary stationary process  $X(t)$ .

#### Discussion

Crepeau and Herzel [1] (CH) have recently compared the spectral entropy  $\sigma$  of Powell and Percival [2] with the standard statistical (Boltzmann–Gibbs–Shannon) entropy  $S$  in three simple physical systems. These two entropies were found to bear little or no resemblance to each other in the systems considered. This was qualitatively interpreted as a reflection of the fact that  $S$  measures static properties while  $\sigma$  measures dynamical properties. Our purpose here is to point out that these two entropies can readily be compared and contrasted in a much more general way by considering their values for an arbitrary stationary process  $X(t)$  with zero mean. This frees the discussion from any reliance on possibly atypical special cases, and brings to the forefront some important general relations between the entropies, time correlations, and probability distributions in  $X$ -space.

The spectral entropy is defined by  $\sigma = -\int_0^\infty d\omega \rho(\omega) \ln \rho(\omega)$ , where  $\rho(\omega)$  is the normalized power spectrum of the process  $X(t)$ . According to the Wiener-Khinchin theorem [3],  $\rho(\omega)$  is simply the normalized Fourier cosine transform of the autocorrelation function  $C(t) = \langle X(0)X(t) \rangle$ ; i.e.,

$$\rho(\omega) = \frac{2}{\pi C(0)} \int_0^\infty dt \cos(\omega t) C(t) \quad (1)$$

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Thus  $\sigma$  is completely determined by  $C(t)$ . The statistical entropy is of course given by  $S = - \int dx P(x) \ln P(x)$ , where  $P(x) = \langle \delta(x - X(t)) \rangle$  is the single-point probability distribution of  $X$ , and the distribution of complete ignorance [4, 5] is presumed to be uniform. (Similarly,  $\sigma$  is implicitly defined relative to a “spectrum of complete ignorance” which is taken to be uniform in  $\omega$ ; i.e., white noise.) Both entropies can therefore be computed from a knowledge of the two-point probability distribution [3]

$$P_2(x_1, x_2, t_1 - t_2) = \langle \delta(x_1 - X(t_1)) \delta(x_2 - X(t_2)) \rangle \quad (2)$$

via the relations  $C(t) = \int dx_1 dx_2 x_1 x_2 P_2(x_1, x_2, t)$  and  $P(x) = \int dx' P_2(x, x', t)$ . These relations show that each entropy discards some of the information in  $P_2$  required to compute the other one, so they clearly cannot be equivalent.

As frequently happens, the situation becomes much simpler when  $X(t)$  is Gaussian. In this case  $P_2$  is completely determined by  $C(t)$  alone [3], and a knowledge of  $C(t)$  is then sufficient to determine both entropies.

Further insight into the essential difference between these two entropies may be obtained by considering the scaled process  $X_{\alpha\beta}(t) = \alpha X(\beta t)$ , in which the amplitude is scaled by the parameter  $\alpha$  while all characteristic time scales are simultaneously scaled by the parameter  $1/\beta$ . Quantities pertaining to this scaled process will be indicated by the subscript  $\alpha\beta$ ; e.g.,  $S_{\alpha\beta}$ . Clearly  $C_{\alpha\beta}(t) = \alpha^2 C(\beta t)$ , which combines with equation (1) to give  $\rho_{\alpha\beta}(\omega) = (1/\beta) \rho(\omega/\beta)$ . It then readily follows that  $\sigma_{\alpha\beta} = \sigma + \ln \beta$ , which is independent of  $\alpha$ . In a similar way, we readily find that  $P_{\alpha\beta}(x) = (1/\alpha) P(x/\alpha)$ , from which it follows that  $S_{\alpha\beta} = S + \ln \alpha$ , which is independent of  $\beta$ . Thus the spectral entropy is independent of the amplitude of the fluctuations but increases logarithmically with their frequency, while the statistical entropy is independent of the frequency of the fluctuations but increases logarithmically with their amplitude. This difference in scaling behavior confirms the qualitative interpretation of CH, but expresses it in a more precise and quantitative way.

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